

# First Four Moments of Metalog Distributions With Up To Ten Terms

This document provides exact expressions for the first four moments of metalog probability distributions with any number of terms from 2 through 10. These moments were calculated and validated with the aid of Wolfram Mathematica by Tom Keelin, August, 2016.

The 10 term metalog quantile function (where  $y$  is cumulative probability,  $0 < y < 1$ ) is:

$$M_{10}(Y) := \left( a_1 + \text{Log} \left[ \frac{Y}{1-Y} \right] a_2 + \left( -\frac{1}{2} + Y \right) \text{Log} \left[ \frac{Y}{1-Y} \right] a_3 + \left( -\frac{1}{2} + Y \right)^2 a_4 + \left( -\frac{1}{2} + Y \right)^2 \text{Log} \left[ \frac{Y}{1-Y} \right] a_5 + \left( -\frac{1}{2} + Y \right)^2 \text{Log} \left[ \frac{Y}{1-Y} \right] a_6 + \left( -\frac{1}{2} + Y \right)^3 a_7 + \left( -\frac{1}{2} + Y \right)^3 \text{Log} \left[ \frac{Y}{1-Y} \right] a_8 + \left( -\frac{1}{2} + Y \right)^4 a_9 + \left( -\frac{1}{2} + Y \right)^4 \text{Log} \left[ \frac{Y}{1-Y} \right] a_{10} \right)$$

## moments notation

- $\mu'_{1,n}$  mean (first moment) of the n term metalog
- $\mu_{2,n}$  variance (2 nd central moment) of the n term metalog
- $\mu_{3,n}$  skewness (3 rd central moment) of the n term metalog
- $\mu_{4,n}$  kurtosis (4 th central moment) of the n term metalog

## 2-term metalog moments

$$\mu'_{1,2} := a_1$$

$$\mu_{2,2} := \frac{1}{3} \pi^2 a_2^2$$

$$\mu_{3,2} := 0$$

$$\mu_{4,2} := \frac{7}{15} \pi^4 a_2^4$$

## 3-term metalog moments

$$\mu'_{1,3} := \mu'_{1,2} + \frac{a_3}{2}$$

$$\mu_{2,3} := \mu_{2,2} + \frac{a_3^2}{12} + \frac{1}{36} \pi^2 a_3^2$$

$$\mu_{3,3} := \mu_{3,2} + \pi^2 a_2^2 a_3 + \frac{1}{24} \pi^2 a_3^3$$

$$\mu_{4,3} := \mu_{4,2} + \frac{3}{2} \pi^2 a_2^2 a_3^2 + \frac{7}{30} \pi^4 a_2^2 a_3^2 + \frac{a_3^4}{80} + \frac{1}{24} \pi^2 a_3^4 + \frac{7 \pi^4 a_3^4}{1200}$$

## 4-term metalog moments

$$\mu'_{1,4} := \mu'_{1,3}$$

$$\mu_{2,4} := \mu_{2,3} + a_2 a_4 + \frac{a_4^2}{12}$$

$$\mu_{3,4} := \mu_{3,3} + \frac{1}{2} a_2 a_3 a_4 + \frac{1}{6} \pi^2 a_2 a_3 a_4 + \frac{1}{8} a_3 a_4^2$$

$$\mu_{4,4} := \mu_{4,3} + 2 \pi^2 a_2^3 a_4 + \frac{1}{2} a_2 a_3^2 a_4 + \frac{2}{3} \pi^2 a_2 a_3^2 a_4 + 2 a_2^2 a_4^2 + \frac{1}{6} \pi^2 a_2^2 a_4^2 + \frac{1}{8} a_3^2 a_4^2 + \frac{1}{40} \pi^2 a_3^2 a_4^2 +$$

$$\frac{1}{3} a_2 a_3^3 + \frac{a_4^4}{80}$$

## 5-term metalog moments

$$\mu'_{1,5} := \mu'_{1,4} + \frac{a_5}{12}$$

$$\mu_{2,5} := \mu_{2,4} + \frac{a_3 a_5}{12} + \frac{a_5^2}{180}$$

$$\mu_{3,5} := \mu_{3,4} + a_2^2 a_5 + \frac{1}{24} a_3^2 a_5 + \frac{1}{180} \pi^2 a_3^2 a_5 + \frac{1}{4} a_2 a_4 a_5 + \frac{1}{60} a_4^2 a_5 + \frac{1}{120} a_3 a_5^2 + \frac{a_5^3}{3780}$$

$$\mu_{4,5} := \mu_{4,4} + a_2^2 a_3 a_5 + \frac{1}{2} \pi^2 a_2^2 a_3 a_5 + \frac{1}{24} a_3^3 a_5 + \frac{1}{40} \pi^2 a_3^3 a_5 + \frac{1}{6} a_2 a_3 a_4 a_5 + \frac{2}{45} \pi^2 a_2 a_3 a_4 a_5 +$$

$$\frac{3}{40} a_3 a_4^2 a_5 + \frac{1}{6} a_2^2 a_5^2 + \frac{1}{90} \pi^2 a_2^2 a_5^2 + \frac{1}{45} a_3^2 a_5^2 + \frac{11 \pi^2 a_3^2 a_5^2}{7560} + \frac{1}{15} a_2 a_4 a_5^2 + \frac{11 a_4^2 a_5^2}{2520} +$$

$$\frac{1}{420} a_3 a_5^3 + \frac{a_5^4}{15120}$$

## 6-term metalog moments

$$\mu'_{1,6} := \mu'_{1,5}$$

$$\mu_{2,6} := \mu_{2,5} + \frac{2 a_2 a_6}{3} + \frac{1}{18} \pi^2 a_2 a_6 + \frac{a_4 a_6}{6} + \frac{a_6^2}{12} + \frac{1}{240} \pi^2 a_6^2$$

$$\mu_{3,6} := \mu_{3,5} + \frac{1}{2} a_2 a_3 a_6 + \frac{5}{12} \pi^2 a_2 a_3 a_6 + \frac{1}{4} a_3 a_4 a_6 + \frac{1}{40} \pi^2 a_3 a_4 a_6 + \frac{1}{3} a_2 a_5 a_6 +$$

$$\frac{1}{90} \pi^2 a_2 a_5 a_6 + \frac{13}{240} a_4 a_5 a_6 + \frac{1}{8} a_3 a_6^2 + \frac{1}{24} \pi^2 a_3 a_6^2 + \frac{3}{80} a_5 a_6^2 + \frac{1}{840} \pi^2 a_5 a_6^2$$

$$\mu_{4,6} := \mu_{4,5} + \frac{8}{3} \pi^2 a_2^3 a_6 + \frac{7}{45} \pi^4 a_2^3 a_6 + \frac{2}{5} a_2 a_2^2 a_6 + \frac{13}{12} \pi^2 a_2 a_2^2 a_6 + \frac{7}{100} \pi^4 a_2 a_2^2 a_6 + 3 a_2^2 a_4 a_6 +$$

$$\pi^2 a_2^2 a_4 a_6 + \frac{1}{4} a_3^2 a_4 a_6 + \frac{17}{120} \pi^2 a_3^2 a_4 a_6 + a_2 a_2^2 a_4 a_6 + \frac{1}{20} \pi^2 a_2 a_2^2 a_4 a_6 + \frac{23}{360} a_4^3 a_6 + \frac{5}{6} a_2 a_3 a_5 a_6 +$$

$$\frac{7}{36} \pi^2 a_2 a_2 a_3 a_5 a_6 + \frac{23}{120} a_3 a_4 a_5 a_6 + \frac{1}{105} \pi^2 a_3 a_4 a_5 a_6 + \frac{17}{180} a_2 a_2^2 a_6 + \frac{11 \pi^2 a_2 a_2^2 a_6}{3780} +$$

$$\frac{1}{70} a_4 a_2^2 a_6 + \frac{6}{5} a_2^2 a_6^2 + \pi^2 a_2^2 a_6^2 + \frac{7}{200} \pi^4 a_2^2 a_6^2 + \frac{1}{8} a_3^2 a_6^2 + \frac{1}{160} \pi^2 a_3^2 a_6^2 + \frac{1}{160} \pi^4 a_3^2 a_6^2 +$$

$$a_2 a_4 a_6^2 + \frac{23}{120} \pi^2 a_2 a_4 a_6^2 + \frac{7}{60} a_2^2 a_4 a_6^2 + \frac{1}{224} \pi^2 a_2^2 a_4 a_6^2 + \frac{7}{60} a_3 a_5 a_6^2 + \frac{211 \pi^2 a_3 a_5 a_6^2}{10080} +$$

$$\frac{67 a_5^2 a_6^2}{6048} + \frac{\pi^2 a_5^2 a_6^2}{3360} + \frac{1}{3} a_2 a_6^3 + \frac{7}{45} \pi^2 a_2 a_6^3 + \frac{1}{240} \pi^4 a_2 a_6^3 + \frac{11}{120} a_4 a_6^3 + \frac{11}{840} \pi^2 a_4 a_6^3 +$$

$$\frac{19 a_6^4}{720} + \frac{409 \pi^2 a_6^4}{45360} + \frac{7 \pi^4 a_6^4}{34560}$$

## 7-term metalog moments

$$\mu'_{1,7} := \mu'_{1,6}$$

$$\mu_{2,7} := \mu_{2,6} + \frac{a_2 a_7}{6} + \frac{a_4 a_7}{40} + \frac{23 a_6 a_7}{720} + \frac{a_7^2}{448}$$

$$\mu_{3,7} := \mu_{3,6} + \frac{1}{4} a_2 a_3 a_7 + \frac{1}{40} \pi^2 a_2 a_3 a_7 + \frac{7}{120} a_3 a_4 a_7 + \frac{7}{120} a_3 a_4 a_7 + \frac{13}{240} a_2 a_5 a_7 + \frac{1}{140} a_4 a_5 a_7 +$$

$$\frac{11}{160} a_3 a_6 a_7 + \frac{1}{224} \pi^2 a_3 a_6 a_7 + \frac{47 a_5 a_6 a_7}{4032} + \frac{29 a_3 a_7^2}{4480} + \frac{a_5 a_7^2}{1344}$$

$$\mu_{4,7} := \mu_{4,6} + a_2^3 a_7 + \frac{1}{3} \pi^2 a_2^3 a_7 + \frac{1}{4} a_2 a_2^2 a_3 a_7 + \frac{17}{120} \pi^2 a_2 a_2^2 a_3 a_7 + a_2^2 a_4 a_7 + \frac{1}{20} \pi^2 a_2^2 a_4 a_7 +$$

$$\frac{19}{240} a_2^2 a_4 a_7 + \frac{1}{112} \pi^2 a_2^2 a_4 a_7 + \frac{23}{120} a_2 a_2^2 a_4 a_7 + \frac{1}{112} a_3 a_7 + \frac{23}{120} a_2 a_3 a_5 a_7 + \frac{1}{105} \pi^2 a_2 a_3 a_5 a_7 +$$

$$\frac{163 a_3 a_4 a_5 a_7}{5040} + \frac{1}{70} a_2 a_2^2 a_5 a_7 + \frac{1}{560} a_4 a_2^2 a_5 a_7 + a_2^2 a_6 a_7 + \frac{23}{120} \pi^2 a_2^2 a_6 a_7 + \frac{43}{480} a_2^2 a_6 a_7 +$$

$$\frac{17}{560} \pi^2 a_2^2 a_6 a_7 + \frac{7}{15} a_2 a_4 a_6 a_7 + \frac{1}{56} \pi^2 a_2 a_4 a_6 a_7 + \frac{11}{280} a_2^2 a_4 a_6 a_7 + \frac{1391 a_3 a_5 a_6 a_7}{30240} +$$

$$\frac{1}{504} \pi^2 a_3 a_5 a_6 a_7 + \frac{239 a_5^2 a_6 a_7}{75600} + \frac{11}{40} a_2 a_2^2 a_6 a_7 + \frac{11}{280} \pi^2 a_2 a_2^2 a_6 a_7 + \frac{409 a_4 a_6^2 a_7}{7560} +$$

$$\frac{1}{576} \pi^2 a_4 a_6^2 a_7 + \frac{359 a_6^3 a_7}{15120} + \frac{563 \pi^2 a_6^3 a_7}{201600} + \frac{7}{60} a_2 a_2^2 a_7 + \frac{1}{224} \pi^2 a_2 a_2^2 a_7 + \frac{1301 a_3^2 a_7^2}{120960} + \frac{\pi^2 a_3^2 a_7^2}{1152} +$$

$$\frac{11}{280} a_2 a_4 a_7^2 + \frac{1}{384} a_4^2 a_7^2 + \frac{81 a_3 a_5 a_7^2}{22400} + \frac{17 a_5^2 a_7^2}{88704} + \frac{409 a_2 a_6 a_7^2}{7560} + \frac{1}{576} \pi^2 a_2 a_6 a_7^2 +$$

$$\frac{563 a_4 a_6 a_7^2}{67200} + \frac{141 a_6^2 a_7^2}{22400} + \frac{\pi^2 a_6^2 a_7^2}{5632} + \frac{563 a_2 a_7^3}{201600} + \frac{a_4 a_7^3}{2816} + \frac{1627 a_6 a_7^3}{2661120} + \frac{a_7^4}{53248}$$

## 8-term metalog moments

$$\mu_{1,8} := \mu_{1,7} + \frac{a_8}{12}$$

$$\mu_{2,8} := \mu_{2,7} + \frac{a_3 a_8}{12} + \frac{1}{120} \pi^2 a_3 a_8 + \frac{13 a_5 a_8}{720} + \frac{a_8^2}{80} + \frac{\pi^2 a_8^2}{1344}$$

$$\mu_{3,8} := \mu_{3,7} + \frac{3}{4} a_2^2 a_8 + \frac{1}{6} \pi^2 a_2^2 a_8 + \frac{1}{24} a_3^2 a_8 + \frac{41 \pi^2 a_3^2 a_8}{1440} + \frac{1}{4} a_2 a_4 a_8 + \frac{1}{40} \pi^2 a_2 a_4 a_8 +$$

$$\frac{13}{480} a_4^2 a_8 + \frac{13}{480} a_3 a_5 a_8 + \frac{1}{420} \pi^2 a_3 a_5 a_8 + \frac{11 a_5^2 a_8}{5040} + \frac{1}{3} a_2 a_6 a_8 + \frac{59}{720} \pi^2 a_2 a_6 a_8 +$$

$$\frac{3}{40} a_4 a_6 a_8 + \frac{1}{224} \pi^2 a_4 a_6 a_8 + \frac{23}{480} a_6^2 a_8 + \frac{59 \pi^2 a_6^2 a_8}{6720} + \frac{3}{40} a_2 a_7 a_8 + \frac{1}{224} \pi^2 a_2 a_7 a_8 +$$

$$\frac{3}{224} a_4 a_7 a_8 + \frac{1153 a_6 a_7 a_8}{60480} + \frac{\pi^2 a_6 a_7 a_8}{1152} + \frac{38400}{160} a_3 a_8^2 + \frac{89 \pi^2 a_3 a_8^2}{13440} + \frac{251 a_5 a_8^2}{60480} +$$

$$\frac{\pi^2 a_5 a_8^2}{4032} + \frac{a_8^3}{448} + \frac{59 \pi^2 a_8^3}{115200}$$

$$\mu_{4,8} := \mu_{4,7} + \frac{9}{10} a_2^2 a_3 a_8 + \frac{7}{6} \pi^2 a_2^2 a_3 a_8 + \frac{7}{100} \pi^4 a_2^2 a_3 a_8 + \frac{1}{24} a_3^3 a_8 + \frac{7}{120} \pi^2 a_3^3 a_8 +$$

$$\frac{1}{240} \pi^4 a_3^3 a_8 + \frac{5}{6} a_2 a_3 a_4 a_8 + \frac{5}{18} \pi^2 a_2 a_3 a_4 a_8 + \frac{23}{240} a_3 a_4^2 a_8 + \frac{1}{112} \pi^2 a_3 a_4^2 a_8 + \frac{5}{12} a_2^2 a_5 a_8 +$$

$$\frac{13}{120} \pi^2 a_2^2 a_5 a_8 + \frac{79 a_3^2 a_5 a_8}{1440} + \frac{101 \pi^2 a_3^2 a_5 a_8}{6048} + \frac{13}{60} a_2 a_4 a_5 a_8 + \frac{1}{105} \pi^2 a_2 a_4 a_5 a_8 +$$

$$\frac{179 a_4^2 a_5 a_8}{10080} + \frac{37 a_3 a_5^2 a_8}{3024} + \frac{\pi^2 a_3 a_5^2 a_8}{1680} + \frac{43 a_5^3 a_8}{75600} + \frac{5}{6} a_2 a_3 a_6 a_8 + \frac{217}{360} \pi^2 a_2 a_3 a_6 a_8 +$$

$$\frac{1}{40} \pi^4 a_2 a_3 a_6 a_8 + \frac{7}{30} a_3 a_4 a_6 a_8 + \frac{103 \pi^2 a_3 a_4 a_6 a_8}{1680} + \frac{49}{180} a_2 a_5 a_6 a_8 + \frac{649 \pi^2 a_2 a_5 a_6 a_8}{15120} +$$

$$\frac{155 a_4 a_5 a_6 a_8}{3024} + \frac{1}{504} \pi^2 a_4 a_5 a_6 a_8 + \frac{11}{80} a_3 a_6^2 a_8 + \frac{1129 \pi^2 a_3 a_6^2 a_8}{15120} + \frac{7 \pi^4 a_3 a_6^2 a_8}{2880} +$$

## 8-term metalog moments (continued)

$$\begin{aligned}
 & \frac{361 a_5 a_6^2 a_8}{10\,080} + \frac{949 \pi^2 a_5 a_6^2 a_8}{201\,600} + \frac{7}{30} a_2 a_3 a_7 a_8 + \frac{103 \pi^2 a_2 a_3 a_7 a_8}{1\,680} + \frac{187 a_3 a_4 a_7 a_8}{3\,780} + \\
 & \frac{1}{288} \pi^2 a_3 a_4 a_7 a_8 + \frac{155 a_2 a_5 a_7 a_8}{3\,024} + \frac{1}{504} \pi^2 a_2 a_5 a_7 a_8 + \frac{263 a_4 a_5 a_7 a_8}{33\,600} + \frac{1979 a_3 a_6 a_7 a_8}{30\,240} + \\
 & \frac{341 \pi^2 a_3 a_6 a_7 a_8}{25\,200} + \frac{1591 a_5 a_6 a_7 a_8}{129\,600} + \frac{\pi^2 a_5 a_6 a_7 a_8}{2\,376} + \frac{839 a_3 a_7^2 a_8}{134\,400} + \frac{\pi^2 a_3 a_7^2 a_8}{2816} + \\
 & \frac{7877 a_5 a_7^2 a_8}{8\,870\,400} + \frac{1}{4} a_2^2 a_8 + \frac{59}{360} \pi^2 a_2^2 a_8 + \frac{1}{160} \pi^4 a_2^2 a_8 + \frac{47 a_3^2 a_8^2}{1\,440} + \frac{67 \pi^2 a_3^2 a_8^2}{2\,688} + \frac{7 \pi^4 a_3^2 a_8^2}{5\,760} + \\
 & \frac{3}{20} a_2 a_4 a_8^2 + \frac{13}{420} \pi^2 a_2 a_4 a_8^2 + \frac{11}{756} a_4^2 a_8^2 + \frac{\pi^2 a_4^2 a_8^2}{1\,152} + \frac{559 a_3 a_5 a_8^2}{30\,240} + \frac{769 \pi^2 a_3 a_5 a_8^2}{201\,600} + \\
 & \frac{2971 a_5^2 a_8^2}{1\,814\,400} + \frac{17 \pi^2 a_5^2 a_8^2}{2\,666\,112} + \frac{8}{45} a_2 a_6 a_8^2 + \frac{11}{140} \pi^2 a_2 a_6 a_8^2 + \frac{7 \pi^4 a_2 a_6 a_8^2}{2\,880} + \frac{11}{280} a_4 a_6 a_8^2 + \\
 & \frac{463 \pi^2 a_4 a_6 a_8^2}{67\,200} + \frac{29 a_6^2 a_8^2}{1\,120} + \frac{1913 \pi^2 a_6^2 a_8^2}{201\,600} + \frac{7 \pi^4 a_6^2 a_8^2}{28\,160} + \frac{11}{280} a_2 a_7 a_8^2 + \frac{463 \pi^2 a_2 a_7 a_8^2}{67\,200} + \\
 & \frac{17 a_4 a_7 a_8^2}{2\,400} + \frac{\pi^2 a_4 a_7 a_8^2}{2816} + \frac{37\,381 a_6 a_7 a_8^2}{3\,628\,800} + \frac{41\,111 \pi^2 a_6 a_7 a_8^2}{2\,661\,120} + \frac{171 a_7^2 a_8^2}{197\,120} + \frac{\pi^2 a_7^2 a_8^2}{26\,624} + \\
 & \frac{29 a_3 a_8^3}{3\,360} + \frac{199 \pi^2 a_3 a_8^3}{44\,800} + \frac{7 \pi^4 a_3 a_8^3}{42\,240} + \frac{6827 a_5 a_8^3}{3\,628\,800} + \frac{7877 \pi^2 a_5 a_8^3}{26\,611\,200} + \frac{43 a_8^4}{57\,600} + \frac{57 \pi^2 a_8^4}{197\,120} + \frac{7 \pi^4 a_8^4}{798\,720}
 \end{aligned}$$

## 9-term metalog moments

$$\mu'_{1,9} := \mu'_{1,8} + \frac{a_9}{80}$$

$$\mu_{2,9} := \mu_{2,8} + \frac{7 a_3 a_9}{360} + \frac{a_5 a_9}{420} + \frac{a_8 a_9}{224} + \frac{a_9^2}{3600}$$

$$\mu_{3,9} := \mu_{3,8} + \frac{1}{4} a_2^2 a_9 + \frac{1}{60} a_3^2 a_9 + \frac{1}{840} \pi^2 a_3^2 a_9 + \frac{7}{120} a_2 a_4 a_9 + \frac{1}{280} a_4^2 a_9 + \frac{5 a_3 a_5 a_9}{1008} +$$

$$\frac{a_5^2 a_9}{4200} + \frac{11}{120} a_2 a_6 a_9 + \frac{1}{420} \pi^2 a_2 a_6 a_9 + \frac{3}{224} a_4 a_6 a_9 + \frac{629 a_6^2 a_9}{60480} + \frac{\pi^2 a_6^2 a_9}{3600} + \frac{3}{224} a_2 a_7 a_9 +$$

$$\frac{1}{600} a_4 a_7 a_9 + \frac{67 a_6 a_7 a_9}{22400} + \frac{9 a_7^2 a_9}{49280} + \frac{559 a_3 a_8 a_9}{60480} + \frac{\pi^2 a_3 a_8 a_9}{1800} + \frac{43 a_5 a_8 a_9}{33600} + \frac{1}{768} a_8^2 a_9 +$$

$$\frac{3 \pi^2 a_8^2 a_9}{49280} + \frac{a_3 a_9^2}{1400} + \frac{19 a_5 a_9^2}{277200} + \frac{9 a_8 a_9^2}{49280} + \frac{a_9^3}{156000}$$

$$\mu_{4,9} := \mu_{4,8} + \frac{1}{2} a_2^2 a_3 a_9 + \frac{7}{60} \pi^2 a_2^2 a_3 a_9 + \frac{1}{60} a_3^3 a_9 + \frac{1}{1680} \pi^2 a_3^3 a_9 + \frac{1}{4} a_2 a_3 a_4 a_9 + \frac{1}{105} \pi^2 a_2 a_3 a_4 a_9 +$$

$$\frac{11}{560} a_3 a_4 a_9 + \frac{1}{10} a_2^2 a_5 a_9 + \frac{1}{210} \pi^2 a_2^2 a_5 a_9 + \frac{97 a_3^2 a_5 a_9}{7560} + \frac{\pi^2 a_3^2 a_5 a_9}{1400} + \frac{43 a_2 a_4 a_5 a_9}{1260} +$$

$$\frac{3 a_4^2 a_5 a_9}{1400} + \frac{1}{540} a_3 a_5^2 a_9 + \frac{a_5^3 a_9}{15400} + \frac{7}{24} a_2 a_3 a_6 a_9 + \frac{41}{840} \pi^2 a_2 a_3 a_6 a_9 + \frac{853 a_3 a_4 a_6 a_9}{15120} +$$

$$\frac{1}{450} \pi^2 a_3 a_4 a_6 a_9 + \frac{199 a_2 a_5 a_6 a_9}{3780} + \frac{1}{700} \pi^2 a_2 a_5 a_6 a_9 + \frac{3}{400} a_4 a_5 a_6 a_9 + \frac{1147 a_3 a_6^2 a_9}{30240} +$$

$$\frac{547 \pi^2 a_3 a_6^2 a_9}{100800} + \frac{563 a_5 a_6^2 a_9}{90720} + \frac{251 \pi^2 a_5 a_6^2 a_9}{1663200} + \frac{853 a_2 a_3 a_7 a_9}{15120} + \frac{1}{450} \pi^2 a_2 a_3 a_7 a_9 +$$

$$\frac{29 a_3 a_4 a_7 a_9}{3360} + \frac{3}{400} a_2 a_5 a_7 a_9 + \frac{251 a_4 a_5 a_7 a_9}{277200} + \frac{449 a_3 a_6 a_7 a_9}{33600} + \frac{3 \pi^2 a_3 a_6 a_7 a_9}{6160} +$$



## 9-term metalog moments (continued)

$$\begin{aligned}
 & \frac{1873 a_5 a_6 a_7 a_9}{1108800} + \frac{4337 a_3 a_7 a_9}{4435200} + \frac{191 a_5 a_7^2 a_9}{1921920} + \frac{37 a_2^2 a_8 a_9}{240} + \frac{3 \pi^2 a_2^2 a_8 a_9}{112} + \frac{221 a_3^2 a_8 a_9}{12096} + \\
 & \frac{451 \pi^2 a_3^2 a_8 a_9}{100800} + \frac{241 a_2 a_4 a_8 a_9}{3780} + \frac{1}{450} \pi^2 a_2 a_4 a_8 a_9 + \frac{23 a_4^2 a_8 a_9}{4800} + \frac{3179 a_3 a_5 a_8 a_9}{453600} + \\
 & \frac{251 \pi^2 a_3 a_5 a_8 a_9}{831600} + \frac{251 a_5^2 a_8 a_9}{554400} + \frac{73 a_2 a_6 a_8 a_9}{840} + \frac{563 \pi^2 a_2 a_6 a_8 a_9}{50400} + \frac{5 a_4 a_6 a_8 a_9}{336} + \\
 & \frac{3 \pi^2 a_4 a_6 a_8 a_9}{6160} + \frac{40217 a_6^2 a_8 a_9}{3628800} + \frac{2377 \pi^2 a_6^2 a_8 a_9}{1900800} + \frac{5 a_2 a_7 a_8 a_9}{336} + \frac{3 \pi^2 a_2 a_7 a_8 a_9}{6160} + \\
 & \frac{3 a_4 a_7 a_8 a_9}{1408} + \frac{11779 a_6 a_7 a_8 a_9}{3326400} + \frac{\pi^2 a_6 a_7 a_8 a_9}{9360} + \frac{19857 a_7^2 a_8 a_9}{81536000} + \frac{21127 a_3 a_8^2 a_9}{3628800} + \\
 & \frac{13787 \pi^2 a_3 a_8^2 a_9}{13305600} + \frac{1247 a_5 a_8^2 a_9}{1330560} + \frac{191 \pi^2 a_5 a_8^2 a_9}{5765760} + \frac{57 a_8^3 a_9}{98560} + \frac{6619 \pi^2 a_8^3 a_9}{81536000} + \frac{11 a_2^2 a_9}{756} + \\
 & \frac{\pi^2 a_2^2 a_9}{1800} + \frac{31 a_3^2 a_9}{16800} + \frac{17 \pi^2 a_3^2 a_9}{184800} + \frac{19 a_2 a_4 a_9}{4200} + \frac{17 a_4^2 a_9}{61600} + \frac{137 a_3 a_5 a_9}{277200} + \frac{2689 a_5^2 a_9}{108108000} + \\
 & \frac{31 a_2 a_6 a_9}{4200} + \frac{17 \pi^2 a_2 a_6 a_9}{92400} + \frac{559 a_4 a_6 a_9}{554400} + \frac{2909 a_6^2 a_9}{3326400} + \frac{37 \pi^2 a_6^2 a_9}{1872000} + \frac{559 a_2 a_7 a_9}{554400} + \\
 & \frac{37 a_4 a_7 a_9}{312000} + \frac{5149 a_6 a_7 a_9}{22422400} + \frac{13 a_7^2 a_9}{985600} + \frac{3349 a_3 a_8 a_9}{3326400} + \frac{37 \pi^2 a_3 a_8 a_9}{936000} + \frac{4129 a_5 a_8 a_9}{33633600} + \\
 & \frac{2747 a_8^2 a_9}{20384000} + \frac{13 \pi^2 a_8^2 a_9}{2956800} + \frac{283 a_3 a_9}{6306300} + \frac{157 a_5 a_9}{36036000} + \frac{a_8 a_9^3}{88704} + \frac{31 a_9^4}{106080000} +
 \end{aligned}$$

## 10-term metalog moments

$$\mu^{1,1,10} := \mu^{1,1,9}$$

$$\mu_{2,10} := \mu_{2,9} + \frac{a_2 a_{10}}{6} + \frac{1}{120} \pi^2 a_2 a_{10} + \frac{23 a_4 a_{10}}{720} + \frac{7 a_6 a_{10}}{180} + \frac{1}{672} \pi^2 a_6 a_{10} + \frac{11 a_7 a_{10}}{1680} + \frac{409 a_{10}^2}{90720} + \frac{\pi^2 a_{10}^2}{6912}$$

$$\mu_{3,10} := \mu_{3,9} + \frac{1}{4} a_2 a_3 a_{10} + \frac{1}{12} \pi^2 a_2 a_3 a_{10} + \frac{11}{160} a_3 a_4 a_{10} + \frac{1}{224} \pi^2 a_3 a_4 a_{10} + \frac{1}{40} a_2 a_5 a_{10} + \frac{3}{40} a_2 a_5 a_{10} +$$

$$\frac{1}{420} \pi^2 a_2 a_5 a_{10} + \frac{47 a_4 a_5 a_{10}}{4032} + \frac{19}{240} a_3 a_6 a_{10} + \frac{39 \pi^2 a_3 a_6 a_{10}}{2240} + \frac{131 a_5 a_6 a_{10}}{7560} + \frac{\pi^2 a_5 a_6 a_{10}}{2016} +$$

$$\frac{521 a_3 a_7 a_{10}}{30240} + \frac{\pi^2 a_3 a_7 a_{10}}{1152} + \frac{49 a_5 a_7 a_{10}}{19200} + \frac{23}{240} a_2 a_8 a_{10} + \frac{59 \pi^2 a_2 a_8 a_{10}}{3360} + \frac{1153 a_4 a_8 a_{10}}{60480} +$$

$$\frac{\pi^2 a_4 a_8 a_{10}}{1152} + \frac{29 a_6 a_8 a_{10}}{1120} + \frac{171 \pi^2 a_6 a_8 a_{10}}{44800} + \frac{313 a_7 a_8 a_{10}}{67200} + \frac{\pi^2 a_7 a_8 a_{10}}{5632} + \frac{629 a_2 a_9 a_{10}}{30240} +$$

$$\frac{\pi^2 a_2 a_9 a_{10}}{1800} + \frac{67 a_4 a_9 a_{10}}{22400} + \frac{13 a_6 a_9 a_{10}}{2688} + \frac{3 \pi^2 a_6 a_9 a_{10}}{24640} + \frac{851 a_7 a_9 a_{10}}{1267200} + \frac{167 a_3 a_{10}^2}{15120} +$$

$$\frac{757 \pi^2 a_3 a_{10}^2}{403200} + \frac{7331 a_5 a_{10}^2}{3628800} + \frac{\pi^2 a_5 a_{10}^2}{19008} + \frac{48911 a_8 a_{10}^2}{14515200} + \frac{281 \pi^2 a_8 a_{10}^2}{665280} + \frac{4313 a_9 a_{10}^2}{7603200} + \frac{\pi^2 a_9 a_{10}^2}{74880}$$

$$\mu_{4,10} := \mu_{4,9} + \frac{4}{5} a_2^3 a_{10} + \frac{2}{3} \pi^2 a_2^3 a_{10} + \frac{7}{300} \pi^4 a_2^3 a_{10} + \frac{1}{4} a_2 a_3^2 a_{10} + \frac{23}{80} \pi^2 a_2 a_3^2 a_{10} +$$

$$\frac{1}{80} \pi^4 a_2 a_3^2 a_{10} + a_2^2 a_4 a_{10} + \frac{23}{120} \pi^2 a_2^2 a_4 a_{10} + \frac{43}{480} a_3^2 a_4 a_{10} + \frac{17}{560} \pi^2 a_3^2 a_4 a_{10} + \frac{7}{30} a_2 a_4^2 a_{10} +$$

$$\frac{1}{112} \pi^2 a_2 a_4 a_{10} + \frac{11}{840} a_3^3 a_{10} + \frac{7}{30} a_2 a_3 a_5 a_{10} + \frac{211 \pi^2 a_2 a_3 a_5 a_{10}}{5040} + \frac{1391 a_3 a_4 a_5 a_{10}}{30240} +$$

$$\frac{1}{504} \pi^2 a_3 a_4 a_5 a_{10} + \frac{67 a_2 a_5^2 a_{10}}{3024} + \frac{\pi^2 a_2 a_5^2 a_{10}}{1680} + \frac{239 a_4 a_5^2 a_{10}}{75600} + a_2^2 a_6 a_{10} + \frac{7}{15} \pi^2 a_2^2 a_6 a_{10} +$$

$$\frac{1}{80} \pi^4 a_2^2 a_6 a_{10} + \frac{1}{10} a_3^2 a_6 a_{10} + \frac{4303 \pi^2 a_3^2 a_6 a_{10}}{60480} + \frac{7 \pi^4 a_3^2 a_6 a_{10}}{2880} + \frac{11}{20} a_2 a_4 a_6 a_{10} +$$

## 10-term metalog moments (continued)

$$\begin{aligned}
 & \frac{11}{140} \pi^2 a_2 a_4 a_6 a_{10} + \frac{409 a_4^2 a_6 a_{10}}{7560} + \frac{1}{576} \pi^2 a_4^2 a_6 a_{10} + \frac{937 a_3 a_5 a_6 a_{10}}{15120} + \frac{929 \pi^2 a_3 a_5 a_6 a_{10}}{100800} + \\
 & \frac{673 a_5^2 a_6 a_{10}}{129600} + \frac{17 \pi^2 a_5^2 a_6 a_{10}}{133056} + \frac{19}{60} a_2 a_6^2 a_{10} + \frac{409 \pi^2 a_2 a_6^2 a_{10}}{3780} + \frac{7 \pi^4 a_2 a_6^2 a_{10}}{2880} + \frac{359 a_4 a_6^2 a_{10}}{5040} + \\
 & \frac{563 \pi^2 a_4 a_6^2 a_{10}}{67200} + \frac{457 a_6^3 a_{10}}{15120} + \frac{47 \pi^2 a_6^3 a_{10}}{5600} + \frac{7 \pi^4 a_6^3 a_{10}}{42240} + \frac{11}{40} a_2^2 a_7 a_{10} + \frac{11}{280} \pi^2 a_2^2 a_7 a_{10} + \\
 & \frac{163 a_3^2 a_7 a_{10}}{6048} + \frac{1339 \pi^2 a_3^2 a_7 a_{10}}{201600} + \frac{409 a_2 a_4 a_7 a_{10}}{3780} + \frac{1}{288} \pi^2 a_2 a_4 a_7 a_{10} + \frac{563 a_4^2 a_7 a_{10}}{67200} + \\
 & \frac{20063 a_3 a_5 a_7 a_{10}}{1814400} + \frac{\pi^2 a_3 a_5 a_7 a_{10}}{2376} + \frac{197 a_5^2 a_7 a_{10}}{277200} + \frac{359 a_2 a_6 a_7 a_{10}}{2520} + \frac{563 \pi^2 a_2 a_6 a_7 a_{10}}{33600} + \\
 & \frac{141 a_4 a_6 a_7 a_{10}}{5600} + \frac{\pi^2 a_4 a_6 a_7 a_{10}}{1408} + \frac{21757 a_6^2 a_7 a_{10}}{1209600} + \frac{1627 \pi^2 a_6^2 a_7 a_{10}}{887040} + \frac{141 a_2 a_7^2 a_{10}}{11200} + \\
 & \frac{\pi^2 a_2 a_7^2 a_{10}}{2816} + \frac{1627 a_4 a_7^2 a_{10}}{887040} + \frac{13063 a_6 a_7^2 a_{10}}{4435200} + \frac{\pi^2 a_6 a_7^2 a_{10}}{13312} + \frac{88069 a_3^2 a_{10}}{645765120} + \frac{11}{40} a_2 a_3 a_8 a_{10} + \\
 & \frac{1129 \pi^2 a_2 a_3 a_8 a_{10}}{7560} + \frac{7 \pi^4 a_2 a_3 a_8 a_{10}}{1440} + \frac{1979 a_3 a_4 a_8 a_{10}}{30240} + \frac{341 \pi^2 a_3 a_4 a_8 a_{10}}{25200} + \\
 & \frac{361 a_2 a_5 a_8 a_{10}}{5040} + \frac{949 \pi^2 a_2 a_5 a_8 a_{10}}{100800} + \frac{1591 a_4 a_5 a_8 a_{10}}{129600} + \frac{\pi^2 a_4 a_5 a_8 a_{10}}{2376} + \frac{211 a_3 a_6 a_8 a_{10}}{2520} + \\
 & \frac{2431 \pi^2 a_3 a_6 a_8 a_{10}}{67200} + \frac{7 \pi^4 a_3 a_6 a_8 a_{10}}{7040} + \frac{33251 a_5 a_6 a_8 a_{10}}{1814400} + \frac{28031 \pi^2 a_5 a_6 a_8 a_{10}}{13305600} + \\
 & \frac{32009 a_3 a_7 a_8 a_{10}}{1814400} + \frac{8047 \pi^2 a_3 a_7 a_8 a_{10}}{26661120} + \frac{4349 a_5 a_7 a_8 a_{10}}{1478400} + \frac{5 \pi^2 a_5 a_7 a_8 a_{10}}{54912} + \\
 & \frac{29}{560} a_2 a_8^2 a_{10} + \frac{1913 \pi^2 a_2 a_8^2 a_{10}}{100800} + \frac{7 \pi^4 a_2 a_8^2 a_{10}}{14080} + \frac{37381 a_4 a_8^2 a_{10}}{3628800} + \frac{4111 \pi^2 a_4 a_8^2 a_{10}}{26661120} +
 \end{aligned}$$

## 10-term metalog moments (continued)

$$\begin{aligned}
 & \frac{81 a_6 a_8^2 a_{10}}{5600} + \frac{421 \pi^2 a_6 a_8^2 a_{10}}{92400} + \frac{7 \pi^4 a_6 a_8^2 a_{10}}{66560} + \frac{5899 a_7 a_8^2 a_{10}}{2217600} + \frac{75329 \pi^2 a_7 a_8^2 a_{10}}{215255040} + \\
 & \frac{1147 a_2 a_3 a_9 a_{10}}{15120} + \frac{547 \pi^2 a_2 a_3 a_9 a_{10}}{50400} + \frac{449 a_3 a_4 a_9 a_{10}}{33600} + \frac{3 \pi^2 a_3 a_4 a_9 a_{10}}{6160} + \frac{563 a_2 a_5 a_9 a_{10}}{45360} + \\
 & \frac{251 \pi^2 a_2 a_5 a_9 a_{10}}{831600} + \frac{1873 a_4 a_5 a_9 a_{10}}{1108800} + \frac{11749 a_3 a_6 a_9 a_{10}}{604800} + \frac{5417 \pi^2 a_3 a_6 a_9 a_{10}}{2217600} + \\
 & \frac{9733 a_5 a_6 a_9 a_{10}}{3326400} + \frac{191 \pi^2 a_5 a_6 a_9 a_{10}}{2882880} + \frac{6073 a_3 a_7 a_9 a_{10}}{1900800} + \frac{\pi^2 a_3 a_7 a_9 a_{10}}{9360} + \frac{211877 a_5 a_7 a_9 a_{10}}{550368000} + \\
 & \frac{40217 a_2 a_8 a_9 a_{10}}{1814400} + \frac{2377 \pi^2 a_2 a_8 a_9 a_{10}}{950400} + \frac{11779 a_4 a_8 a_9 a_{10}}{3326400} + \frac{\pi^2 a_4 a_8 a_9 a_{10}}{9360} + \\
 & \frac{1237 a_6 a_8 a_9 a_{10}}{221760} + \frac{254827 \pi^2 a_6 a_8 a_9 a_{10}}{448448000} + \frac{569629 a_7 a_8 a_9 a_{10}}{672672000} + \frac{\pi^2 a_7 a_8 a_9 a_{10}}{42240} + \\
 & \frac{2909 a_2 a_9 a_{10}}{1663200} + \frac{37 \pi^2 a_2 a_9 a_{10}}{936000} + \frac{5149 a_4 a_9 a_{10}}{22422400} + \frac{1254479 a_6 a_9 a_{10}}{3027024000} + \frac{13 \pi^2 a_6 a_9 a_{10}}{1478400} + \\
 & \frac{30463 a_7 a_9 a_{10}}{576576000} + \frac{19 a_2^2 a_{10}^2}{120} + \frac{409 \pi^2 a_2^2 a_{10}^2}{7560} + \frac{7 \pi^4 a_2^2 a_{10}^2}{5760} + \frac{997 a_3^2 a_{10}^2}{60480} + \frac{6949 \pi^2 a_3^2 a_{10}^2}{806400} + \\
 & \frac{7 \pi^4 a_3^2 a_{10}^2}{28160} + \frac{359 a_2 a_4 a_{10}^2}{5040} + \frac{563 \pi^2 a_2 a_4 a_{10}^2}{67200} + \frac{141 a_4^2 a_{10}^2}{22400} + \frac{\pi^2 a_4^2 a_{10}^2}{5632} + \frac{29069 a_3 a_5 a_{10}^2}{3628800} + \\
 & \frac{27431 \pi^2 a_3 a_5 a_{10}^2}{26611200} + \frac{146429 a_5^2 a_{10}^2}{239500800} + \frac{83 \pi^2 a_5^2 a_{10}^2}{5930496} + \frac{457 a_2 a_6 a_{10}^2}{5040} + \frac{141 \pi^2 a_2 a_6 a_{10}^2}{5600} + \\
 & \frac{7 \pi^4 a_2 a_6 a_{10}^2}{14080} + \frac{21757 a_4 a_6 a_{10}^2}{1209600} + \frac{1627 \pi^2 a_4 a_6 a_{10}^2}{887040} + \frac{3737 a_6^2 a_{10}^2}{302400} + \frac{13063 \pi^2 a_6^2 a_{10}^2}{4435200} +
 \end{aligned}$$

## 10-term metalog moments (continued)

$$\begin{aligned}
 & \frac{7 \pi^4 a_6^2 a_{10}^2}{133120} + \frac{21757 a_2 a_7 a_{10}^2}{1209600} + \frac{1627 \pi^2 a_2 a_7 a_{10}^2}{887040} + \frac{13063 a_4 a_7 a_{10}^2}{4435200} + \frac{\pi^2 a_4 a_7 a_{10}^2}{13312} + \\
 & \frac{1421 a_6 a_7 a_{10}^2}{316800} + \frac{88069 \pi^2 a_6 a_7 a_{10}^2}{215255040} + \frac{4193641 a_7^2 a_{10}^2}{12108096000} + \frac{\pi^2 a_7^2 a_{10}^2}{122880} + \frac{17477 a_3 a_8 a_{10}^2}{1451520} + \\
 & \frac{115697 \pi^2 a_3 a_8 a_{10}^2}{26611200} + \frac{7 \pi^4 a_3 a_8 a_{10}^2}{66560} + \frac{1108007 a_5 a_8 a_{10}^2}{479001600} + \frac{1387601 \pi^2 a_5 a_8 a_{10}^2}{5811886080} + \\
 & \frac{941011 a_8^2 a_{10}^2}{479001600} + \frac{79314709 \pi^2 a_8^2 a_{10}^2}{145297152000} + \frac{7 \pi^4 a_8^2 a_{10}^2}{614400} + \frac{65479 a_3 a_9 a_{10}^2}{26611200} + \frac{2241643 \pi^2 a_3 a_9 a_{10}^2}{8072064000} + \\
 & \frac{12577183 a_5 a_9 a_{10}^2}{36324288000} + \frac{73 \pi^2 a_5 a_9 a_{10}^2}{9884160} + \frac{202589633 a_8 a_9 a_{10}^2}{290594304000} + \frac{16397 \pi^2 a_8 a_9 a_{10}^2}{251596800} + \\
 & \frac{47711 a_9^2 a_{10}^2}{968647680} + \frac{101 \pi^2 a_9^2 a_{10}^2}{101836800} + \frac{3737 a_2 a_{10}^3}{453600} + \frac{13063 \pi^2 a_2 a_{10}^3}{6652800} + \frac{7 \pi^4 a_2 a_{10}^3}{199680} + \frac{1421 a_4 a_{10}^3}{950400} + \\
 & \frac{88069 \pi^2 a_4 a_{10}^3}{645765120} + \frac{130751 a_6 a_{10}^3}{59875200} + \frac{4193641 \pi^2 a_6 a_{10}^3}{9081072000} + \frac{7 \pi^4 a_6 a_{10}^3}{921600} + \frac{8968709 a_7 a_{10}^3}{24216192000} + \\
 & \frac{1423 \pi^2 a_7 a_{10}^3}{46126080} + \frac{20662987 a_{10}^4}{145297152000} + \frac{264263 \pi^2 a_{10}^4}{9686476800} + \frac{7 \pi^4 a_{10}^4}{16711680} + \frac{16711680}{16711680}
 \end{aligned}$$